

## **Using an MKT Developmental Framework to Guide Instruction in Our K-8 Mathematics Specialization Program**

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### Abstract

Illinois State University holds a unique position in its ability to educate mathematics teachers at all levels. This paper outlines the courses used in developing mathematics specialists for the K-8 classroom and discusses a framework for examining student learning and preparedness for teaching mathematics. The framework focuses on Mathematical Knowledge for Teaching (MKT), describing levels of development from emerging to deep and connected MKT. The framework consists of four separate but inter-related components, each important to the overall development of connected knowledge.

Illinois State University is uniquely positioned within mathematics education. It has the largest secondary mathematics teacher education program in the State of Illinois, as well as one of the three largest in the country. The Mathematics Department also houses an undergraduate mathematics specialist program for elementary and middle-level teachers, as well as Masters and PhD programs in mathematics education.

While there are a variety of programs that span the K-16 range, we focus here on the elementary and middle-level specialist program, which consists of specifically designed courses in mathematics content and methods for preservice and inservice teachers. The goal of the program is to increase the mathematical knowledge for teaching (MKT) in order to improve the teaching and learning of mathematics at the K-8 level. This paper provides an overview of the courses comprising the program, as well as a discussion of the framework of indicators of MKT that we have developed. This framework gives us a basis for reassessing what we do in the individual courses and helps to identify weaknesses in our students and in the program as a whole.

### Our elementary and middle-level programs

The Department has developed a specific set of mathematics content courses for the

elementary and middle-level mathematics specialist program (see figure 1). While many of the courses have texts that are used as a basis, every course also includes supplemental problems and activities which we have developed to further students' mathematical knowledge for teaching.

**Figure 1. Illinois State University's Elementary & Middle-Level Math Specialization**

<b>Course # &amp; Credit Hours</b>	<b>Course Title &amp; Text Reference</b>	<b>Who Takes This Course?</b>
MAT 130 3 credit hours	<i>Dimensions of Numerical Reasoning</i>  No text	All elementary education majors and any K-9 mathematics specialists. Also serves as an inner core mathematics course for general education.
MAT 131 3 credit hours	<i>Geometric Reasoning: Geometry as Earth Measures</i>  Kinsey & Moore (2002)	A middle core general education course, <i>suggested</i> for all elementary education majors. Required of all elementary education and middle level mathematics specialists.
MAT 152 3 credit hours	<i>Number Structures II</i> Lappan et al.(1997/1998)	All elementary education majors NOT specializing in mathematics.
MAT 201 3 credit hours	<i>Teaching Mathematics in the Elementary School</i>  Carpenter et al (1999)	All elementary education majors.
MAT 202 4 credit hours	<i>Algebraic Reasoning for K-8 Teachers</i>  No text	All elementary and middle-level math specialists
MAT 309 4 credit hours	<i>Number Theory for K-8 Teachers</i>  Gross & Harris (2004)	All elementary and middle-level math specialists
MAT 312 4 credit hours	<i>Probabilistic &amp; Statistical Reasoning for K-8 Teachers</i>  Perkowski & Perkowski (2007)	All elementary and middle-level math specialists
MAT 304 4 credit hours	<i>Modern Geometry for 5-8 Teachers</i>  Kay (2001)	All middle level math specialists. A required course for master-level students in K-9 math ed.
MAT 307 4 credit hours	<i>Abstract Algebra for Middle School Teachers</i>  Nicodemi, Surtherland & Towsley (2007)	One of three capstone electives for undergraduate middle level math specialists. An elective for master-level students.
MAT 314 4 credit hours	<i>Concepts of Calculus for Middle School Teachers</i>  Harcharras & Mitrea (2001)	One of three capstone electives for undergraduate middle level math specialists.
MAT 315 4 credit hours	<i>Mathematical Modeling for Middle School Teachers</i>  Davis & Edwards (2007)	One of three capstone electives for undergraduate middle level math specialists. An elective for master-level students.
MAT 302 4 credit hours	<i>Teaching Mathematics in Grades 6-8</i>  Boaler & Humphreys (2005) Langrall, Meier & Mooney (2008)	All elementary and middle level math specialists. An elective for master-level students.

These courses start with a general education course called the Dimensions of Numerical Reasoning, which is required of all elementary majors regardless of their intent to specialize in mathematics. The course focuses on rational number sense and understanding the operations and procedures they memorized in the past. It is a problem-based course that requires students to think about numerical relationships in new ways. We have not found a text that is suitable for this course, so we have created a large set of problems that specifically identify and address students' misconceptions. Students are required to purchase overhead manipulative sets that they use to show relationships and operations. Teachers require students to solve problems in multiple ways, fully explain and justify their procedures, and to make connections among their representations.

The program then continues with a second general education course on Geometric Reasoning, which approaches the study of geometry from its historical underpinnings as "earth measures". This course includes interdisciplinary and real-world connections and uses of geometry from a measurement perspective. In addition to the text, students must use Geometer's Sketchpad and other resources to complete several projects. These projects range from drawing a campus building in one and two perspectives, using projective geometry techniques to keep depths proportional, to using transformational geometry to analyze and create Escher-like tessellations. Do to the project work of this nature, the course has also been popular with Art and Theater majors, who struggle to find general education mathematics courses they find worthwhile.

After the general education courses, the students specializing in K-8 mathematics will take courses in Algebraic Reasoning, Number Theory, and Probability and Statistics. Each of these courses are again, specifically designed for the K-8 teachers focusing on developing deeper understandings of the topics commonly taught at the K-8 level and addressing misconceptions that are common among these students.

The Algebraic Reasoning course currently has no text. Like the Dimensions of Numerical Reasoning course, we have developed a large set of tasks that require students to develop a deeper understanding of algebra by exploring the uses of literal symbols, generalizing patterns, modeling data, and consistently connecting numerical, symbolic, graphical, and tabular representations of relationships.

The Number Theory course includes a lengthy study of numerical relationships involving primes, composites, multiples, and factors. It includes work with the Euclidean Algorithm to help solve Diophantine equations, as well as work with modular arithmetic. Again, the book provides a starting point for much of this work, but we have developed a set of problem-solving tasks to supplement the book and focus on the students misconceptions that develop when they focus on the procedures.

The Probabilistic and Statistical Reasoning course is designed to engage the students in a variety of mathematics activities that address the content taught at the middle school level and the underlying higher-level content. The course emphasizes statistical literacy, uses real-world data, and stresses conceptual understanding rather than just

knowledge of procedures. Computer and calculator technology is used throughout the course. Topics include organizing and displaying data, using numbers to describe data, data with two variables, probability, counting techniques, random variables and probability distributions. Students explore activities that enhance their understanding of the material and complete a set of extra problems as well as a project that helps them apply the content learned to real-world situations as well as the classroom.

The students desiring to work specifically at the middle level will take an additional course in Modern Geometry and one of three capstone content courses. While the Modern Geometry course provides a solid core of formal proof based on axiomatic systems in both Euclidean and at least one non-Euclidean model of geometry, it also contains a major component that connects geometry and algebra through matrix representations of transformations. The capstone course options include Concepts of Calculus, Mathematical Modeling, or Abstract Algebra. Each of the capstone courses has content that is more closely aligned with a traditional mathematics course at the college level, but the order and structure is often modified to more easily draw the connections back to the K-8 mathematics they will be teaching. This allows the preservice teachers to see how the content taught at the K-8 level provides the underpinnings of even these more advanced mathematical topics.

Finally, the Mathematics Department also provides all the methods courses associated with the elementary and middle-level mathematics programs. This includes an elementary methods course, an undergraduate course in middle-level mathematics instruction, as well as a special instructional methods course for middle-level inservice teachers. Each of the undergraduate method courses involves a clinical component where students visit local schools to work with K-8 students in mathematics classrooms. We assign preservice teachers to work with one or two other preservice teachers to plan and implement lessons for small groups of K-8 students. During these lessons the faculty methods instructor watches the small groups and takes notes about each groups' activities and the middle school students' responses to the lesson. Afterwards, these groups of preservice teachers are required to visit with their instructor and reflect on the lesson in a guided reflection session. Preservice teachers are provided some generic guidelines to reflect on prior to the meeting, but the instructor will ask additional questions specific to his/her observations as well as probe the preservice teachers' thinking further about the reflections they have done prior to the session. This ensures that both the methods instructor and the preservice teachers get to address issues and concerns they have about the lessons taught.

Because of the nature of these courses, each of these courses is taught by tenured or tenure-line mathematics education faculty members, who focus on the use of multiple representations and connections. Students are required to work together to explore the content, explain their thinking, and, through in-depth discussions, address their own misconceptions while developing a deeper conceptual understanding of the content. Even so, individual faculty can differ on the exact ways we present content and even vary on the

exact content presented in various sections of the same class. While minor differences are to be expected, the general principles and intent of each course must be maintained to ensure the integrity of the program.

As a result we have begun working on ways to examine our instruction and develop a more unified goal of producing the best elementary and middle-level mathematics teachers that we can. As we considered this goal, it seemed natural to start with Ball, Thames, & Phelps (2007) work on the mathematical knowledge for teaching which includes: the notions of common mathematical content knowledge, specialized mathematical content knowledge, knowledge of mathematical content and students, knowledge of mathematical content and mathematics curricula, and horizon level mathematical knowledge. As we considered these various types of mathematical knowledge and how we should go about producing these in our preservice teachers, we recognized the need for a theoretical framework that describes characteristics of preservice teachers as they develop this deep and connected mathematical knowledge for teaching. A multiple year study of student work ensued, which resulted in the Meier and Rich MKT Framework.

#### Overview of the Meier & Rich MKT Framework

The Meier & Rich MKT Framework was developed by examining student work across many undergraduate and graduate mathematics content and methods classes. Student work and tasks were carefully analyzed using a grounded theory approach to find the common components and characteristics representing student development of MKT.

The resulting Meier & Rich Framework of indicators includes four components of deep and connected mathematical knowledge for teaching, and provides a five-level progression of indicators that demonstrate growth for each component. The four components we have identified as central to growth in MKT were the students' ability to: explain and justify their work, use multiple representations, recognize and generalize relationships among conceptually similar problems, and pose problems. Each component begins with an entry level of knowledge that is characterized by common content knowledge and procedures expected of any mathematically literate person, and builds to Level 4 which represents deep and connected mathematical knowledge for teaching (see Figure 2).

**Figure 2. Meier & Rich progression of indicators describing preservice teachers’ levels of development of deep & connected mathematical knowledge for teaching.**

Level→ <i>Components</i> ↓	Entry level (common content knowledge & procedures)	Level 1 (Emerging)	Level 2 (Developing)	Level 3 (Maturing)	Level 4 (Deep & Connected MKT)
<i>Ability to correctly solve a task, explain their work, justify their reasoning &amp; make connections.</i>	Students show some work or show detailed step-by-step work in symbol form without explanations or with explanations that just discuss what or how they solved the problem without any discussion of why the methods were appropriate.	Students start to show work and describe more than just what they did, but their discussion of why is limited to either generalities, without connection to the specifics of the problem, or only highlights the specifics and doesn't relate them to general mathematical concepts that are present.	Students show their work and can discuss why some of their procedures were appropriate, but usually only after they have completed the solution process. They can discuss how an appropriate math concept was related to the specifics of this task, but only after they have completed the task. The students at this point can usually create informal proofs or justifications for solutions.	Students show their work and include justifications of their solutions process as they solve the problem. They can clearly tell you what they are doing and why they are doing it as they solve the problem, rather than having to wait until afterwards. At this point, students are usually able to judge other students' complete solutions and justifications. They may be able to create more formal proofs at this time.	Students can analyze the problem and the specifics of the task, identify and discuss the concepts involved in the task without actually solving the problem. The concepts necessary for solving the problem and justifying the solution, as well as potentially useful representations, are recognized and evaluated for their connections to other relevant concepts or problems prior to solving the task.
<i>Ability to use multiple representation</i>	Students can occasionally translate a problem into another form, but often use only their own non-standard representations which may or may not be appropriate.	Students can work with a single standard representation. Students are still inconsistent in their ability to translate a problem from one standard form to another.	Students can usually work with more than a single standard representation, but they work with them in parallel, completing each as a separate task. Students can consistently translate the same problem into multiple equivalent forms.	Students can work with multiple standard representations simultaneously, while making connections between the various forms. Students may be unable to productively respond when presented with someone else's non-standard representations.	Students can evaluate and judge the accuracy of multiple standard representations as students are using them, and they are able to productively respond to other's use of non-standard mathematical representations.
<i>Ability to recognize, use and generalize relationships among conceptually similar problems</i>	Students do not recognize patterns among problems, solving each as if they are unrelated tasks, and changing anything about the structure of the presentation will cause students difficulty. Even fairly trivial changes are problematic.	Students may be able to use patterns to solve problems, but have difficulty generalizing that pattern. Students will struggle with non-trivial structural or contextual changes in the problems.	Students may be able to generalize patterns in problems that include minor structural and contextual changes. Students will still struggle with major changes in constructs, and generalizing solutions to problems with more than one possible answer.	Students can generalize patterns when asked, as well as make connections between conceptually similar problems which involve different constructs or contexts. Students are able to look for patterns in multiple solutions and write a generalized form of the solution when one is possible.	Students begin to generalize solutions, whenever appropriate, without being prompted. Students are able to generate problems or tasks that will produce patterns that exhibit desired properties for generalization in multiple contexts.
<i>Ability to pose problems</i>	Students have difficulty writing word problems or tasks that involve the required concepts. Students attempt to replicate problems already seen, but cannot make even trivial changes without difficulty, and will change numbers or contexts without thought to whether these changes produce situations which are not feasible.	Students can only write problems that use the same concept by starting with a given problem and changing the numbers, while leaving other words and contexts relatively unchanged. Most changes attempted are fairly trivial, and when they attempt non-trivial changes they often change the concepts in the problem without realizing it.	Students can consistently write similar problems using other appropriate terminology or contexts. The changes made from the original start to become more nontrivial without causing difficulties. However, students still rarely attempt to make major structural or contextual changes. The students still have trouble writing appropriate extension problems that will build on the concept present in the original task.	Students begin to successfully write conceptually similar tasks with major structural or contextual changes. Students can also extend a given task to advance the concepts beyond those present in the original problem. Students may not always be able to judge the level of extension present in their new task. It may be a small step beyond the original or require a large amount of additional connections, and the student does not always recognize which type of extension they have written, resulting in relatively simple extensions, or problems that are too complex to be appropriate.	Students can extend tasks appropriately to develop rich connections. Students begin to efficiently select or adapt appropriate tasks to highlight concepts or connections for classroom use. Students begin to develop a repertoire of questions to ask themselves to help identify the existence of connections between ideas, and questions to ask others to help identify weaknesses in understanding of various concepts.

The framework is not linear, and the levels do not represent discrete jumps in ability. Students do not move strictly in one direction along a component row of indicators. Students will often loop back, as content areas change or the content progresses to more sophisticated or abstract concepts. Students also oscillate between levels before moving more consistently to the higher level. Students are rarely at the same level in all components simultaneously, unless they have already achieved deep and connected MKT. A student can be at level 2 in his/her ability to explain and justify and remain at the entry level in his/her ability to pose problems. However, one of the most striking findings in our development of the framework was the presence of an “anchoring” effect. That is, if one component area had not progressed, it hindered the advancement in other components as well. For example, students may have progressed in their ability to explain and justify, reaching level 2, but if they were unable to pose problems above an entry level, they stagnated in their ability to explain or justify as well and did not easily advance to level 3. So while we see the four components as separate, they are interconnected. This interconnectedness is clearly seen in students who possess deep and connected MKT.

#### Uses of the framework

The Meier & Rich Framework has given the department a new lens through which to examine our courses and instruction. We are now able to look at individual classroom activities and assess how they are helping to improve a student’s MKT. In the past we have known the mathematical concepts being developed with various activities, but we sometimes struggled to see how these activities were helping students develop their MKT as well as their content knowledge. The manner in which the courses were structured was based on a constructivist approach to learning, but each course was focused specifically on the development of appropriate mathematical ideas, and often seemed to miss the mark in developing deeper connections needed for teaching-related content. The framework provides a clearer picture of the development of various components of deeper connected understanding that teachers need and helps instructors ensure they address all these components in the courses. In this manner we are building consistency of focus among all the faculty members teaching various content and methods courses.

We see this framework as having the potential for having a wide impact on mathematics education courses, both at our institution and others. Our students rarely make it above level 3 consistently upon completion of their undergraduate program with many still remaining at level 2. By consistently addressing each of the components, we would hope to improve that result. Additionally, we would like to look at our master’s level courses in mathematics education from the standpoint of this framework to ensure that practicing teachers enrolled in that program continue to grow toward a deeper and connected MKT.

Practicing teachers who have examined the framework have told us that they appreciate having some kind of criteria to use in self evaluation, which is concrete enough to

inform them of both their strengths and weaknesses. By using the framework to inform staff development programs, teachers and administrators would be able to have specific goals to work toward that will indicate development in MKT, rather than just having vague goals of improving their understanding or their teaching.

Finally, the framework could help inform policy makers of the characteristics possessed by “highly qualified” teachers of mathematics rather than simply focusing on the courses they have completed or tests they have passed. The levels in the framework could not only help redefine what it means to highly qualified to teach mathematics, but it also provides levels that could help provide evidence of growth toward that goal.

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