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**Content Knowledge and Pedagogical Content Knowledge of Algebra Teachers and Changes in Both Types of Knowledge as a Result of Professional Development**

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Abstract

In seeking to improve the mathematics education of all students, it is important to understand the connection between the content knowledge and pedagogical content knowledge of mathematics and how professionals can influence growth in both of these types of knowledge. We do not have an answer about the interplay of content knowledge and pedagogical content knowledge in successful instructional practices in the mathematics classroom. This study involves assessing the content knowledge and pedagogical content knowledge of secondary teachers of Algebra I. In addition, how are these types of knowledge expressed in instructional practices? Last how do content knowledge, pedagogical content knowledge, and instructional practices change as a result of professional development which gives attention to increasing both types of knowledge?

Algebra I serves as a gateway course in dividing students into classes with significantly different opportunities to learn resulting in differences for future success in more advanced mathematics courses (RAND, 2003), to college preparation (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for the preparation of the world of work (Silver, 1997). Teachers play a key role in ensuring that all students have the opportunities and experiences needed to learn mathematics (Mewborn, 2003).

What knowledge do Algebra I teachers need to possess in order to ensure all students have equitable opportunities to learn Algebra I? Limited research has been conducted in the areas of teacher content knowledge and pedagogical content knowledge of elementary teachers and have shown that teacher's content knowledge is often thin and inadequate to provide the instructional opportunities needed for students to successfully learn mathematics (Ball, 1998a, 2003b, Ball & Bass, 2000; Fuller, 1996, Ma, 1999, Mewborn, 2001, Stacy, et al., 2001). Relatively few studies have focused on the content

knowledge of secondary mathematics teachers, perhaps because of the belief that content knowledge may not be a problem at the secondary level because of secondary teachers' specialized knowledge of mathematics (Ball et al., 2001). However, the available research, while limited, has served to reveal the fallacy of this assumption (Ball et al., 2001). At the secondary level, studies have considered teachers' content knowledge and pedagogical content knowledge in the areas of slope (Stump, 1997; Sherin, 2002) and functions (Even, 1993; Llinares, 2000; Sherin, 2002). Nathan & Koedinger (2000) studied teachers' perceptions about algebraic reasoning.

This paper relates to a study which focused on studying the content knowledge and pedagogical content knowledge of secondary teachers of Algebra I and how both types of knowledge changed as a result of professional development. In addition, changes in instructional practices resulting from changes in these types of knowledge were considered.

### *Content Knowledge and Pedagogical Content Knowledge*

From the review of related literature, how others viewed content knowledge and pedagogical content knowledge was reviewed and synthesized definitions were developed for both types of knowledge. These definitions were used as a backdrop on which the results of this study were analyzed.

*Content Knowledge.* Content knowledge was defined as both the procedural and conceptual knowledge as well as the mathematical processes for using mathematics. Mathematical processes (NCTM, 2000) include the ability to solve mathematical problems in a variety of ways, reasoning and proof, the ability to communicate mathematics, the ability to make connections between mathematical topics and ideas, the ability to use different representations, and the ability to use technology in solving mathematics problems.

*Pedagogical Content Knowledge.* Pedagogical content knowledge was defined as the ability a teacher has to use their knowledge of mathematics to “unwrap” the mathematical topics and present the content in ways for students to successfully learn mathematics (Ma, 1999). This knowledge includes the teachers' ability to use content knowledge to access different representations, different methods for solving mathematical problems, and to help students make connections between mathematical topics as well as making connections between the different representations for solving mathematics problems. In addition, pedagogical content knowledge includes the teachers' ability to recognize errors in student thinking and the ability to pose questions and respond productively to questions from students.

### *Methodology*

This study consisted of two complementary investigations. First, survey research was conducted with a large pool of teachers in order to understand both their content

knowledge and pedagogical content knowledge. Second, a multi-case study was used to provide an in-depth examination of these same types of knowledge, as well as to probe how teachers use their knowledge in mathematical instruction. The cases also offered opportunities to understand how content knowledge and pedagogical content knowledge changed as a result of professional development and to see how these changes are reflected in mathematics instructional practices.

*Subject Selection.* Multi-District Mathematics Systematic Improvement Program (MDMSIP) is a partnership between two universities and twelve school districts in East Alabama. The goal of MDMSIP is to improve mathematics education within the partnership districts. Teachers from nine secondary schools, who were accepted to be included in the summer professional development training as part of Cohort I of MDMSIP, along with an additional school that volunteered to be a part of the baseline data collection of MDMSIP, were used as subjects for the survey research.

*Algebra Content Knowledge Instrument (ALCKIN).* There was not an identifiable instrument to measure content knowledge and pedagogical content knowledge of secondary school teachers, so one had to be developed. In order to develop test items for the survey research, consideration was given to the types of knowledge that teachers should possess in order to teach algebra to high school students along with the major algebraic topics teachers are expected to teach their students. Five key sources were consulted. First the CBMS Report (2000) which includes recommended topic areas for the preparation of teachers. Second, the RAND Report (2003) and Principles and Standards for School Mathematics (2000) contain the big ideas in relationship to the types of algebraic knowledge in which students should be proficient. Third, the Alabama Course of Study and Curriculum Guide for the NSF funded Multi-District Systematic Mathematics Improvement Project (MDSMIP) which provided specific objectives to be covered in the algebra courses at the secondary level. The keys areas of content knowledge focused on were families of functions, using algebraic structures in relationship to expressions, equations, and inequalities, analyzing change in various contexts, using algebraic reasoning in relationship to other mathematical fields, and properties of number systems.

Test development for the instrument began by considering the items from other assessments. The items in content instruments developed by the Learning Mathematics for Teaching (LMT) project (Hill, Schilling, & Ball, 2003) were multiple-choice in format and provided the opportunity for the researcher to see how teachers could solve problems that arise in the classroom. Additional understanding of teachers' mathematical knowledge would be gained by asking participants to explain why they selected particular answer choices. Thus, the quantitative summaries possible with closed-ended items were merged with the deeper insights provided by open-ended responses.

A pool of thirty-five algebra items covering the identified content areas was developed for consideration to be used on the ALCKIN. Some of the mathematics

problems were drawn from National Assessment of Educational Practices [NAEP] (NCES, 2003b), RAND (2003), PSSM (NCTM, 2000), Stump (1997), Llinares (2000), and the LMT project (Hill, Schilling, & Ball, 2003). Additional items were developed to address areas of algebra content that were not covered by these items.

The initial pool of items was field tested to ensure the items were not confusing and to assure the instrument could be completed in a thirty-minute timeframe. The ALCKIN was field tested with seven students who were either undergraduate or doctoral students in mathematics education. All of the doctoral students had previously taught mathematics at the secondary level. Time was recorded as well as comments that were made about anything that was found confusing in the wording of problems. Successive revisions were made until the instrument was of an appropriate length, with well-designed tasks addressing the identified areas.

Sixty-five teachers involved with the MDMSIP gave consent to participate and were administered the ALCKIN. First, analysis of the results from the ALCKIN included correctness of their answers. Second, data from the written explanations was entered in Atlas, ti (Muhr, 1991) which is an analysis program used with qualitative research. Coding of the explanations included codes which were specific to particular test items. For example, the code "Passes vertical line test" may have only been used in Question one which contained graphs. Other codes may have related to more than one question. For example, "Drawings" may have been used as a reason for how students could show two algebraic expressions are equivalent or the participant may have actually used a drawing to clarify their explanation. After a code list was established, each document was revisited to ensure coding was appropriate. Each question was analyzed and conclusions were drawn for that particular question then overall conclusions were made for the instrument.

Figure 1 is question five from the ALCKIN. One area of consideration in the ALCKIN was using algebraic structures in relationship to expressions, equations, and inequalities. The emphasis for the fifth problem on the ALCKIN was not on testing the participants' ability to solve algebraic equations, but rather to give the participants an opportunity to find the solutions for an equation in a format that may be different from the usual method to which teachers and students may be accustomed. The participants were told that a teacher had asked his students to solve the quadratic equation  $3x^2 = 4 - 2x$  using a spreadsheet. This particular problem was selected with the intent purpose that the resulting polynomial not be factorable and that the solution would involve a radical. A spreadsheet table was presented in which a range of values for  $x$  had been substituted in the expressions  $3x^2$  and  $4-2x$ . This table shows that the approximate solution for the quadratic equation should be between -1.5 and -1.6. Note that there is a second solution between 0.8 and 0.9, which is not shown in the table.

5. Mr. Casteel is using spreadsheets in his Algebra class to find solutions for quadratic equations. What approximate solution(s) for the equation  $3x^2 = 4-2x$  should Mr. Casteel's students give using the following spreadsheet?

x	$3x^2$	$4-2x$
-1.8	9.72	7.6
-1.7	8.67	7.4
-1.6	7.68	7.2
-1.5	6.75	7
-1.4	5.88	6.8
-1.3	5.07	6.6
-1.2	4.32	6.4
-1.1	3.63	6.2
-1	3	6
-0.9	2.43	5.8
-0.8	1.92	5.6
-0.7	1.47	5.4
-0.6	1.08	5.2
-0.5	0.75	5
-0.4	0.48	4.8

Solution(s):

Explain your answer.

Figure 1. Question five from the Algebra Content Knowledge Instrument.

*Cases.* Selection of teachers for the cases was made on information obtained from the Alabama Department of Education website related to student achievement on the Alabama High School Graduation Examination [ASHGE] (ASDE, 2003). Initial classroom observations were made of eight teachers who all taught Algebra I, Algebra IA, or Algebra IB. Two of the teachers were selected from the highest achieving school, two from the lowest achieving school, and two teachers each from schools falling between the schools with the highest and lowest achieving students. A further narrowing of cases occurred when two teachers of these eight were not going to be rehired for the upcoming academic year, one who was not going to participate in the Cohort I summer professional development, and a choice was made between two teachers, one who taught both Algebra I and Algebra IA.

Data sources included the ALCKIN, classroom observations, teacher interviews, field notes, researcher journal, and the Reformed Teaching Observation Protocol (RTOP). The RTOP was developed by the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT) at the Arizona State University (Sawade et al., 2000) and the purpose of this instrument was to assess the degree to which the instructional practices of observed mathematics teachers is "reformed." All these types of data collection were used prior to professional development to establish the content knowledge, pedagogical content knowledge, and instructional practices of the cases prior

to professional development as well as after professional development to establish any changes that occurred in both types of knowledge as a result of professional development. Data sources from the cases were also entered into Atlas ti (Muh, 1991). The codes developed in the survey study were used as a beginning point for coding the case study documents. New codes were added to this list of codes as the text from the documents was analyzed. Additional codes were also added to capture events that may normally be observed in classroom observations but may not be a part of written instruments. For example, student conversation was a part of classroom observations and codes, such as “Student Gives Correct Answer” or “Student Indicates They Don’t Understand” had to be included. All of the documents had been coded; each document was revisited to ensure consistency of coding throughout all of the documents.

### *Conclusion*

The conclusion will first focus on the content knowledge and pedagogical content knowledge teachers had prior to professional development. Second, the type of instructional practices used by the cases will be examined. Last, changes in content knowledge and pedagogical content knowledge after professional development as well as changes in instructional practices will be addressed.

*Content Knowledge and Pedagogical Content Knowledge.* On parts of the ALCKIN that required a selection of answers, only seven of the twenty-five tasks were answered correctly by more than eighty percent of the participants. The ability to do mathematics procedurally was predominant in explanations and in the procedures suggested in clarifying answer choices. The majority of the cases exhibited strong procedural knowledge and performed additional mathematical tasks using procedures.

However, there were errors made in the procedures used by some of the participants. This was true for all the items on the ALCKIN. This included computational errors and errors in mathematical notations. The majority of the cases did not make procedural errors.

The conceptual knowledge of all the participants in both studies was limited. A limited view of conceptual knowledge was exhibited by using drawings or algebra tiles or through the suggestion of using manipulatives. Gaps were exhibited in the understanding of topics on the ALCKIN, such as understanding of algebraic expressions, functions, and slope.

Participants in both studies had a difficult time in providing different representations for mathematical situations. Some of the participants did not recognize that algebraic expressions, although equivalent, cannot be used to describe different interpretations by students. Less than half of the participants could explain the meaning of an algebraic expression within the context of an algebraic equation. In addition, the majority of participants did not provide or use multiple methods for solving mathematical situations, and they tended to be quite procedural in nature. When the cases were pushed

to provide other ways of solving mathematical tasks, even alternative procedural methods could not be provided.

In considering the pedagogical content knowledge, first was the ability to “unwrap” mathematics topics and present them so students can be successful in learning mathematics evident? Actual mathematical instruction was not used in answering this question; however, if participants lack an understanding of mathematics topics it could be argued that they do not have the ability for unwrapping them and presenting them in ways so students can be successful.

Next, do teachers have the ability to access different representations as well as methods for solving mathematics problems that arise within mathematics instruction? Participants had a difficult time in solving the spreadsheet problem on the ALCKIN. The different representations of slope also caused a majority of the participants’ problems in recognizing they were equally valid. The cases were able to use their pedagogical content knowledge in analyzing non-procedural methods used by hypothetical students within problem solving context during interviews.

Did the participants have the ability to recognize student errors and be able to respond to them with alternative models and explanations? Participants had difficulty in assessing student errors on the ALCKIN. Instead of responding to errors or statements made by students, participants generally ignored them or responded with how the student should have worked the problem using a set algebraic manipulation, rather than providing alternative explanations or models.

Did the participants have the ability to respond to questions, and to pose questions and problems that are productive to students learning mathematics? While the design of the ALCKIN did not directly address this issue, none of the cases exhibited this particular part of pedagogical content knowledge in interviews or in classroom observations.

Overall, it can be stated teachers in this study had content knowledge. However, their knowledge was primarily procedural and they had limited conceptual knowledge. Also lacking was their ability to use various representations as well as use different methods to solve problems. We can claim that the participants did not have a deep understanding of the algebra content. Participants also exhibited limited pedagogical content knowledge. This was evident in all areas related to the definition of pedagogical content knowledge. This results of this study were similar to studies done with elementary teachers (Ball, 1998a, 2003b, Ball & Bass, 2000, Fuller, 1996, Ma, 1999, Mewborn, 2001, Stacy, et al., 2001), algebra teachers have inadequate content knowledge and pedagogical content knowledge for successful mathematics instruction.

*Content Knowledge and Pedagogical Content Knowledge in Classroom Instruction.* Procedural knowledge was predominant within instructional practice. Cases demonstrated use of procedures within a variety of mathematics topics in almost all of the classroom observations. Three of the cases were not observed making procedural mistakes, while one case did. However, when we consider their conceptual knowledge, it

was seen to be very limited. There was not an occasion that could be identified where the cases exhibited that they knew why rules and algorithms worked. Cases were not observed using different methods of working problems. Conjectures were not investigated, nor was the cases' ability to develop and evaluate arguments observed either. Teachers did communicate ideas and clarify what they meant, but it was in superficial ways related to providing procedures and explaining what to do next. Cases were not observed making connections between mathematical topics. Representations of mathematics did not go beyond the procedures for doing mathematics.

When considering the pedagogical content knowledge, no unwrapping of mathematical topics or ideas was observed, only presentations of particular procedures. It was not obvious that teachers accessed different representations as well as different methods for solving mathematics problems. While teachers may have felt that their students were using alternative methods, in actuality they were generally the same procedures used by the teacher, but using a different order of steps or omitting some steps altogether. Since the teacher was dominant in mathematics instruction, little opportunity was given for other types of mathematical problems to arise that would necessitate different representations.

Connections between mathematical topics were not noted and since the cases did not use, or have students use, different representations for mathematical topics, this facet of pedagogical content knowledge was not present either. Analysis of student errors within the context of mathematical instruction generally related to incorrect mathematical computation errors, the teacher recognizing incorrect answers in algebraic expressions or equations, or using incorrect procedures in completely algebraic tasks. All student errors related to the mathematical topics covered during a particular day's instructions. Neither did results show teachers offering alternative explanations or models.

Cases could respond to questions their students asked during instruction, but the questions were generally about the correctness of an answer, if a student was doing the work correctly, or what the student should do next in solving the mathematical task. Moreover, teachers did ask their students questions productive in successful learning of mathematics, but they generally related to asking students to give short answers to arithmetic problems, the simplification of algebraic expressions, or to name the next step in a procedural process. Therefore, once again, the type of questions required of a teachers' pedagogical content knowledge was limited since these questions were not the type that would be productive to students learning mathematics in the ways they need to.

*Changes in Knowledge and Instructional Practices.* Changes in the procedural knowledge of the cases were not observed. However, changes in conceptual knowledge were evident and were expressed either by demonstration and/or in conversation. One of the cases demonstrated the use of algebra tiles in showing the addition of algebraic expressions while another case drew pictures of algebra tiles and a "magic square" in

solving a quadratic problem. Both of these examples show that the cases could solve problems in different ways and could communicate their understanding clearly. They were also able to make connections to algebraic rules for factoring and it was obvious that they used different representations in demonstrating they understood how to solve the quadratic problem using different methods. All of the cases were able to suggest other non-procedural ways of solving the quadratic equation. Two of them offered conceptual ways of solving the problem.

Growth in pedagogical content knowledge was also evident. One case presented the use of algebra tiles in ways she said she used them in instructional practices. She provided the connections to using algebra tiles, which included using the tiles, drawing pictures, and bridging to the algorithms related to their use. Another case also demonstrated she was using algebra tiles for the multiplication of binomials and factorization. Although instruction was not observed using algebra tiles, it was evident that students had learned with them from the work they did on the board.

Conversations with one of the cases included such terminology as “in-out” tables and how they could be used before introducing terms such as function, domain, and range. She also indicated that she had used them during instruction.

Strictly limiting how changes in content knowledge and pedagogical content knowledge were manifested in classroom instruction through classroom observations, the answer would be simple. Classroom observations of teachers’ instructional practices before and after professional development showed very little difference in their content knowledge and pedagogical content knowledge. It can be noted that one of the cases was using an activity to help students develop the conceptual understanding for factoring the difference of two squares, but even in this one instance, she controlled the classroom conversation and limited the opportunities for her students to really develop the understanding they needed. However, when we consider the types of observations, such as the demonstration of algebra tiles with the two cases and the conversation about “in-out” tables, we might be able to suggest that it was possibly reflected in their instructional practice just not on the days of observation. Also, the display of student work from the IMP units (Fendel et al., 2000) would suggest that changes in both of these types of knowledge are reflected in the cases instructional practices. These are limited views of pedagogical content knowledge, because they do not get everything the definitions of these types of knowledge entail. Furthermore, without these observations within the classroom context, we cannot say that it is used as a tool in helping students make sense of the mathematics or as just another procedure.

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