A Mathematician and Mathematics Educator Collaborate in the Preparation of Prospective Middle Grades Mathematics Teachers

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Our University

- MTSU was originally founded as a state normal school
- We now have the largest student body in the state
- Teacher preparation follows several paths
  - Early Childhood Education, PK -3
  - Elementary Education, K-6
  - Middle Grades, 4-8 with a subject area concentration
    - State licensure is general rather than content specific
  - Secondary Education, Content specific & housed in academic departments

Our middle grades program

- Colleagues on our campus found there were less than a dozen 4-8 majors with a mathematics or science concentration
- The Teachers Now grant was written and funded
- Goals of the grant include:
  - provide scholarships for 4-8 students with a mathematics or science concentration
  - develop relevant mathematics, science, and education courses for these students
  - promote field experiences for these students with exemplary mentor teachers in the local county school system

Discrete Mathematics for Middle Grades Teachers

- One new course developed, but the only one team taught with a mathematician, Dr. D. Nelson, and a mathematics educator, yours truly

- Our goals for the course included:
  - Supporting students' developing understandings of the course topics
  - Providing a setting where students 'formally' and informally presented their original mathematical activity to their classmates and teachers
  - Developing students' Mathematics Knowledge for Teaching (MKT)
  - Providing opportunities for prospective teachers to engage middle grades students in activities requiring discrete mathematics

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Topics and Problems presented

- Topics of the course included sets, logic, counting techniques, probability theory and graph theory
- Problems we posed for pairs of students to present
  - Circular Permutations
  - Expected Value
  - Conceptualizing Large Numbers
  - Nine others
- The usefulness of these problems to support our collaboration

Elements of our Collaboration

- Assumptions about mathematicians and mathematics educators
- Our planning (prior to and during the course)
  - Goal setting
  - Writing problems
  - Preparing exams
  - Assessing students
- Students' perceptions of our collaboration

RULES OF INFERENCE

1. ***Modus Ponens*** **(M.P.)**

$p \to q$
p
$\therefore q$

2. ***Modus Tollens*** **(M.T.)**

$p \to q$
$\sim q$
$\therefore \sim p$

3. ***Hypothetical Syllogism*** **(H.S.)**

$p \to q$
$q \to r$
$\therefore p \to r$

4. ***Disjunctive Syllogism*** **(D.S.)**

$p \vee q$
$\sim p$
$\therefore q$

5. ***Constructive Dilemma*** **(C.D.)**

$(p \to q) \wedge (r \to s)$
$p \vee r$
$\therefore q \vee s$

6. ***Absorption*** **(Abs.)**

$p \to q$
$\therefore p \to (p \wedge q)$

7. ***Simplification*** **(Simp.)**

$p \wedge q$
$\therefore p$

8. ***Conjunction*** **(Conj.)**

p
q
$\therefore p \wedge q$

9. ***Addition*** **(Add.)**

p
$\therefore p \vee q$

Rule of Replacement

Logically equivalent expressions can replace each other wherever they occur.

10.	**De Morgan's Laws (D.M.)**	$\sim(p \wedge q) \equiv (\sim p \vee \sim q)$ $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$
11.	**Commutative Law (Comm.)**	$(p \vee q) \equiv (q \vee p)$ $(p \wedge q) \equiv (q \wedge p)$
12.	**Associative Law (Assoc.)**	$[(p \wedge q) \wedge r] \equiv [p \wedge (q \wedge r)]$ $[(p \vee q) \vee r] \equiv [p \vee (q \vee r)]$
13.	**Distributive Law (Dist.)**	$[p \wedge (q \vee r)] \equiv [(p \wedge q) \vee (p \wedge r)]$ $[p \vee (q \wedge r)] \equiv [(p \vee q) \wedge (p \vee r)]$
14.	**Double Negation (D.N.)**	$\sim\sim p \equiv p$
15.	**Contrapositive (Cont.)**	$(p \to q) \equiv (\sim q \to \sim p)$
16.	**Material Implication (Impl.)**	$(p \to q) \equiv (\sim p \vee q)$
17.	**Material Equivalence (Equiv.)**	$(p \leftrightarrow q) \equiv [(p \to q) \wedge (q \to p)]$ $(p \leftrightarrow q) \equiv [(p \wedge q) \vee (\sim q \wedge \sim p)]$
18.	**Exportation (Exp.)**	$[(p \wedge q) \to r] \equiv [p \to (q \to r)]$
19.	**Tautology (Taut.)**	$p \equiv (p \vee p)$ $p \equiv (p \wedge p)$ $p \equiv (p \wedge \mathcal{T})$ $p \equiv (p \vee \mathcal{F})$

Three Laws of Thought

These statements are tautologies.

20.	**Principle of Identity (P.I)**	$p \to p$
21.	**Principle of Contradiction (P.C.)**	$\sim(p \wedge \sim p)$
22.	**Principle of Excluded Middle (P.E.M.)**	$p \vee \sim p$

23.	**Adjunction (Adj.)**	$[(p \to q) \wedge (p \to r)] \equiv [(p \to (q \wedge r)]$
24.	**Contradiction (Contra.)**	$(p \wedge \sim q) \to \mathcal{F}$ $\therefore p \to q$ $\sim p \to \mathcal{F}$ $\therefore p$
25.	**Proof by Cases**	$[(p \vee r) \to q] \equiv [(p \to q) \wedge (r \to q)]$

the Teachers Now

What we got out of this effort

- Don – perspectives from students who want to make sense of what they are learning
- Don – rationale for thinking about MKT; students being prompted to express their understandings and make sense of these
- Jake – the interconnectedness of these topics and ways we might sequence them in courses
- More about what it takes to collaborate

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Concluding

- Issues to consider
- Questions you all have